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Homogeneous Anisotropic Cosmological Models with Variable Gravitational and Cosmological "Constants"

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The Einstein field equations with perfect fluid source and variable Λ and G for Bianchi-type universes are studied under the assumption of a power-law time variation of the expansion factor, achieved via a suitable power-law assumption for the Hubble parameter suggested by M. S. Berman. All the models have a power-law variation of pressure and density and are singular at the epoch $t = 0$. The variation of $G(t)$ as $1/t$ and $\Lambda(t)$ as $1/t^2$ is consistent with these models.

1. INTRODUCTION

The "cosmological constant problem" can be expressed as the discrepancy between the negligible value Λ has for the present universe (Weinberg, 1972) and the values 10^{50} times larger expected by the Glashow-Salam-Weinberg model (Abers and Lee, 1973) or by the grand unified theory (GUT) (Langacker, 1981), where it should be 10^{107} times larger. Recently Wahba (1989) studied the cosmological function $\Lambda(t)$ in detail. Chen and Wu (1990) suggested that $\Lambda \propto 1/R^2$, where $R(t)$ is the scale factor in the Robertson-Walker model. Abdel-Rahman (1990) considered a model with the same kind of variation. Berman *et al.* (1989), Berman and Som (1990*a*,*b*), and Bertolami (1986*a*,*b*) stressed that the relation $\Lambda \propto t^{-2}$ plays an important role in cosmology. It has been shown by Berman (1983) and Berman and Gomide (1988) that all the phases of the universe, i.e., radiation, inflation, and pressure-free, may be considered as particular cases of the deceleration parameter $q =$ const type, where

$$q = -R\ddot{R}/\dot{R}^2 \tag{1.1}$$

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where dots stand for time derivatives. We extend this definition to the Bianchi-type cosmological models. We consider Einstein's field equations with time-varying Λ and G and take the energy-momentum tensor of a perfect fluid. We assume that the conservation law for matter holds.

2. FIELD EQUATIONS

Einstein's field equations with variable cosmological and gravitational "constants" Λ and G are given by

$$R^{\mu}{}_{\nu} - \tfrac{1}{2}\delta^{\mu}{}_{\nu}R = 8\pi G(t)T^{\mu}{}_{\nu} + \Lambda(t)\delta^{\mu}{}_{\nu} \tag{2.1}$$

where $R^{\mu}{}_{\nu}$ is the Ricci tensor; $R = g^{\mu\nu}R_{\mu\nu}$ is the Ricci scalar; and $T^{\mu}{}_{\nu}$ is the matter energy-momentum tensor.

From the divergence of (2.1), we get

$$8\pi G_{,\mu}T^{\mu}{}_{\nu} + 8\pi G(T^{\mu}{}_{\nu;\mu}) + \Lambda_{,\mu}\delta^{\mu}{}_{\nu} = 0 \tag{2.2}$$

The energy-momentum tensor is

$$T_{\mu\nu} = (\rho + p)u_{\mu}u_{\nu} - pg_{\mu\nu} \tag{2.3}$$

The four-velocity vector u^{μ} is

$$u^{\mu} = [0, 0, 0, (g_{44})^{-1/2}] \tag{2.4}$$

3. BIANCHI TYPE I MODEL

The Bianchi type I metric is

$$dS^2 = dt^2 - R_1^2(t)\,dx^2 - R_2^2(t)\,dy^2 - R_3^2(t)\,dz^2 \tag{3.1}$$

For the metric (3.1), the field equations (2.1) and (2.2) reduce to

$$\frac{\ddot{R}_2}{R_2} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_2\dot{R}_3}{R_2R_3} = 8\pi Gp - \Lambda \tag{3.2}$$

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_1\dot{R}_3}{R_1R_3} = 8\pi Gp - \Lambda \tag{3.3}$$

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\dot{R}_1\dot{R}_2}{R_1R_2} = 8\pi Gp - \Lambda \tag{3.4}$$

$$\frac{\dot{R}_1\dot{R}_2}{R_1R_2} + \frac{\dot{R}_2\dot{R}_3}{R_2R_3} + \frac{\dot{R}_3\dot{R}_1}{R_3R_1} = -8\pi G\rho - \Lambda \tag{3.5}$$

$$8\pi\dot{G}\rho + 8\pi G\left[\dot{\rho} + (\rho + p)\left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3}\right)\right] + \dot{\Lambda} = 0 \tag{3.6}$$

If we suppose the energy conservation law $T^{\mu}{}_{\nu;\mu}=0$ to hold, then (3.6) reduces to

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{R}_1}{R_1}+\frac{\dot{R}_2}{R_2}+\frac{\dot{R}_3}{R_3}\right)=0 \tag{3.7a}$$

$$\dot{\Lambda}=-8\pi\dot{G}\rho \tag{3.7b}$$

where the quantities with dots refer to their derivatives with respect to coordinate t.

We define the 3-volume by

$$V(t)=[R_1R_2R_3]^{1/3} \tag{3.8}$$

We assume the solution of equations (3.2)-(3.7) in the form

$$\begin{aligned} V(t)&=(mDt)^{1/m}\\ R_1(t)&=(m_1D_1t)^{1/m_1}\\ R_2(t)&=(m_2D_2t)^{1/m_2}\\ R_3(t)&=(m_3D_3t)^{1/m_3}\\ \Lambda(t)&=\Lambda_0t^{-2}, \qquad m, m_1, m_2, m_3\neq 0 \end{aligned} \tag{3.9}$$

where m, m_1, m_2, m_3, D, D_1, D_2, D_3, and Λ_0 are arbitrary constants. From (3.8) and (3.9), we get

$$\frac{1}{m}=\frac{1}{3}\left(\frac{1}{m_1}+\frac{1}{m_2}+\frac{1}{m_3}\right) \tag{3.10}$$

Using (3.9) in (3.2) and (3.5), we get the pressure and density respectively,

$$8\pi Gp=\frac{1}{t^2}\left(\Lambda_0+\frac{1}{m_2^2}-\frac{1}{m_2}+\frac{1}{m_3^2}-\frac{1}{m_3}+\frac{1}{m_2m_3}\right) \tag{3.11}$$

$$8\pi G\rho=-\frac{1}{t^2}\left(\Lambda_0+\frac{1}{m_1m_2}+\frac{1}{m_2m_3}+\frac{1}{m_3m_1}\right) \tag{3.12}$$

From equations (3.2)-(3.4) and (3.9), we have

$$\frac{1}{m_2^2}-\frac{1}{m_2}+\frac{1}{m_2m_3}=\frac{1}{m_1^2}-\frac{1}{m_1}+\frac{1}{m_1m_3} \tag{3.13}$$

$$\frac{1}{m_3^2}-\frac{1}{m_3}+\frac{1}{m_2m_3}=\frac{1}{m_1^2}-\frac{1}{m_1}+\frac{1}{m_1m_2} \tag{3.14}$$

From (3.7b) and (3.8), we get

$$8\pi\rho\dot{G}=\frac{\Lambda_0}{t^3} \tag{3.15}$$

Equations (3.12) and (3.15) give $(\dot{G}/G)$ varying as $1/t$. Then G, p, and ρ vary as $1/t$. The model is singular at $t=0$, and with its evolution, the pressure, density, and the cosmological term decrease.

Further,

$$\rho+p=\frac{1}{8\pi G t^2}\left(\frac{1}{m_2^2}-\frac{1}{m_2}+\frac{1}{m_3^2}-\frac{1}{m_3}-\frac{1}{m_1 m_2}-\frac{1}{m_1 m_3}\right) \tag{3.16a}$$

$$\rho+p=\frac{1}{8\pi G t^2}\left(\frac{1}{m_2}-\frac{1}{m_2^2}+\frac{1}{m_3}-\frac{1}{m_3^2}-\frac{1}{m_1 m_2}-\frac{2}{m_2 m_3}-\frac{1}{m_3 m_1}-2\Lambda_0\right) \tag{3.16b}$$

$$\rho+3p=\frac{1}{8\pi G t^2}\left(\frac{3}{m_2^2}-\frac{3}{m_2}+\frac{3}{m_3^2}-\frac{3}{m_3}+\frac{2}{m_2 m_3}-\frac{1}{m_1 m_2}-\frac{1}{m_1 m_3}+2\Lambda_0\right) \tag{3.16c}$$

$$\rho-3p=\frac{1}{8\pi G t^2}\left(\frac{3}{m_2}-\frac{3}{m_2^2}+\frac{3}{m_3}-\frac{3}{m_3^2}-\frac{4}{m_2 m_3}-\frac{1}{m_1 m_2}-\frac{1}{m_1 m_3}-4\Lambda_0\right) \tag{3.16d}$$

The reality conditions $\rho\geq 0$, $p\geq 0$, and $\rho-3p\geq 0$ impose further restrictions on the model besides (3.10), (3.13), and (3.14).

4. BIANCHI TYPE II MODEL

The Bianchi type II metric is

$$dS^2=dt^2-S^2\,dx^2-R^2\,dy^2 \\ -(R^2y^2+\tfrac{1}{4}S^2y^4)\,dz^2-S^2y^2\,dx\,dz, \tag{4.1}$$

where $S=S(t)$ and $R=R(t)$.

The field equations (2.1) and (2.2) for the metric (4.1) lead to

$$2\frac{\ddot{R}}{R}+\left(\frac{\dot{R}}{R}\right)^2-\frac{3}{4}\frac{S^2}{R^4}=8\pi G p-\Lambda \tag{4.2}$$

$$\frac{\ddot{R}}{R}+\frac{\ddot{S}}{S}+\frac{\dot{R}\dot{S}}{RS}+\frac{1}{4}\frac{S^2}{R^4}=8\pi G p-\Lambda \tag{4.3}$$

$$2\frac{\dot{R}\dot{S}}{RS}+\left(\frac{\dot{R}}{R}\right)^2-\frac{1}{4}\frac{S^2}{R^4}=-8\pi G\rho-\Lambda \tag{4.4}$$

$$8\pi\rho\dot{G}+8\pi G\left[\dot{\rho}+(\rho+p)\left(\frac{\dot{S}}{S}+2\frac{\dot{R}}{R}\right)\right]+\dot{\Lambda}=0 \tag{4.5}$$

If we assume that the energy conservation law for matter holds, then (4.5) reduces to

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{S}}{S}+2\frac{\dot{R}}{R}\right)=0 \tag{4.6a}$$

$$\dot{\Lambda}=-8\pi\rho\dot{G} \tag{4.6b}$$

We define

$$V(t)=(SR^2)^{1/3} \tag{4.7}$$

We assume the solution of equations (4.2)-(4.6) in the form

$$\begin{aligned} V(t)&=(mDt)^{1/m}\\ S(t)&=(m_1D_1t)^{1/m_1}\\ R(t)&=(m_2D_2t)^{1/m_2}\\ \Lambda(t)&=\Lambda_0t^{-2}, \qquad m, m_1, m_2\neq 0 \end{aligned} \tag{4.8}$$

where m, m_1, m_2, D, D_1, D_2, Λ_0 are arbitrary constants. From (4.7) and (4.8) we get

$$m=\frac{3m_1m_2}{2m_1+m_2} \tag{4.9}$$

Using (4.8) in (4.2) and (4.4), we get p and ρ, respectively:

$$8\pi Gp=\frac{1}{t^2}\left(\Lambda_0+\frac{3}{m_2^2}-\frac{2}{m_2}\right)-\frac{3}{4}\frac{(m_1D_1t)^{2/m_1}}{(m_2D_2t)^{4/m_2}} \tag{4.10}$$

$$8\pi G\rho=-\frac{1}{t^2}\left(\Lambda_0+\frac{2}{m_1m_2}+\frac{1}{m_2^2}\right)+\frac{1}{4}\frac{(m_1D_1t)^{2/m_1}}{(m_2D_2t)^{4/m_2}} \tag{4.11}$$

From (4.2), (4.3), and (4.8), we have

$$\begin{aligned} &\frac{1}{t^2}\left(\frac{2}{m_2^2}-\frac{1}{m_2}-\frac{1}{m_1^2}+\frac{1}{m_1}-\frac{1}{m_1m_2}\right)\\ &\quad=\frac{(m_1D_1t)^{2/m_1}}{(m_2D_2t)^{4/m_2}} \end{aligned} \tag{4.12}$$

This is satisfied, leading to a relation between the constants, if

$$\frac{2}{m_2}=1+\frac{1}{m_1} \tag{4.13}$$

From (4.6b) and (4.8), we have

$$8\pi\rho\dot{G}=\frac{\Lambda_0}{t^3} \tag{4.14}$$

Equations (4.11) and (4.14) give $\dot{G}/G$. When (4.13) is satisfied, $\dot{G}/G$ varies as $1/t$. Then G, p, and ρ vary as $1/t$. The model is singular at $t=0$.

Further we can easily obtain

$$\rho+p=\frac{1}{8\pi G}\left[\frac{2}{t^2}\left(\frac{1}{m_2^2}-\frac{1}{m_2}-\frac{1}{m_1m_2}\right)-\frac{1}{2}\frac{(m_1D_1t)^{2/m_1}}{(m_2D_2t)^{4/m_2}}\right] \tag{4.15a}$$

$$\rho - p = \frac{1}{8\pi G}\left[-\frac{2}{t^2}\left(\Lambda_0 + \frac{1}{m_1 m_2} + \frac{2}{m_2^2} - \frac{1}{m_2}\right) + \frac{(m_1 D_1 t)^{2/m_1}}{(m_2 D_2 t)^{4/m_2}}\right] \tag{4.15b}$$

$$\rho + 3p = \frac{1}{8\pi G}\left[\frac{2}{t^2}\left(\Lambda_0 + \frac{1}{m_1 m_2} + \frac{4}{m_2^2} - \frac{3}{m_2}\right) - 2\frac{(m_1 D_1 t)^{2/m_1}}{(m_2 D_2 t)^{4/m_2}}\right] \tag{4.15c}$$

$$\rho - 3p = \frac{1}{8\pi G}\left[\frac{2}{t^2}\left(\frac{3}{m_2} - \frac{1}{m_1 m_2} - \frac{5}{m_2^2} - 2\Lambda_0\right) + \frac{5}{2}\frac{(m_1 D_1 t)^{2/m_1}}{(m_2 D_2 t)^{4/m_2}}\right] \tag{4.15d}$$

The reality conditions $\rho \geq 0$, $p \geq 0$, and $\rho - 3p \geq 0$ impose further restrictions on the model besides (4.9), (4.12), and (4.13).

5. BIANCHI TYPE III MODEL

The Bianchi type III metric is

$$dS^2 = dt^2 - R_1^2(t)\, dr^2 - R_2^2(t)[d\theta^2 + \sinh^2\theta\, d\phi^2] \tag{5.1}$$

For the metric (5.1), the field equations (2.1) and (2.2) reduce to

$$2\frac{\ddot{R}_2}{R_2} + \left(\frac{\dot{R}_2}{R_2}\right)^2 - \frac{1}{R_2^2} = 8\pi G p - \Lambda \tag{5.2}$$

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} = 8\pi G p - \Lambda \tag{5.3}$$

$$2\frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} + \left(\frac{\dot{R}_2}{R_2}\right)^2 - \frac{1}{R_2^2} = -8\pi G\rho - \Lambda \tag{5.4}$$

$$8\pi\rho\dot{G} + 8\pi G\left[\dot{\rho} + (\rho + p)\left(\frac{\dot{R}_1}{R_1} + 2\frac{\dot{R}_2}{R_2}\right)\right] + \dot{\Lambda} = 0 \tag{5.5}$$

If we assume that the energy conservation law holds, then (5.5) reduces to

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{R}_1}{R_1} + 2\frac{\dot{R}_2}{R_2}\right) = 0 \tag{5.6a}$$

$$\dot{\Lambda} = -8\pi\dot{G}\rho \tag{5.6b}$$

We define

$$V(t) = (R_1 R_2^2)^{1/3} \tag{5.7}$$

We assume the solution of equations (5.2)-(5.6) in the form

$$\begin{aligned} V(t) &= (mDt)^{1/m} \\ R_1(t) &= (m_1 D_1 t)^{1/m_1} \\ R_2(t) &= (m_2 D_2 t)^{1/m_2} \\ \Lambda(t) &= \Lambda_0 t^{-2}, \qquad m, m_1, m_2 \neq 0 \end{aligned} \tag{5.8}$$

where m, m_1, m_2, D, D_1, D_2, and Λ_0 are arbitrary constants.

From (5.7) and (5.8), we get

$$m = \frac{3m_1 m_2}{2m_1 + m_2} \tag{5.9}$$

Using (5.8) in (5.2) and (5.4), we get p and ρ, respectively:

$$8\pi G p = \frac{1}{t^2}\left(\Lambda_0 + \frac{3}{m_2^2} - \frac{2}{m_2}\right) - \frac{1}{(m_2 D_2 t)^{2/m_2}} \tag{5.10}$$

$$8\pi G \rho = -\frac{1}{t^2}\left(\Lambda_0 + \frac{1}{m_2^2} + \frac{2}{m_1 m_2}\right) + \frac{1}{(m_2 D_2 t)^{2/m_2}} \tag{5.11}$$

From (5.2), (5.3), and (5.8), we get

$$\frac{1}{t^2}\left(\frac{2}{m_2^2} - \frac{1}{m_2} - \frac{1}{m_1^2} + \frac{1}{m_1} - \frac{1}{m_1 m_2}\right) = \frac{1}{(m_2 D_2 t)^{2/m_2}} \tag{5.12}$$

Equation (5.12) is satisfied and leads to a relation between the constants when

$$m_2 = 1 \tag{5.13}$$

From (5.6b) and (5.8), we have

$$4\pi\rho\dot{G} = \frac{\Lambda_0}{t^3} \tag{5.14}$$

Equations (5.11) and (5.14) give $\dot{G}/G$ when (5.13) is satisfied,

$$\frac{\dot{G}}{G} \propto \frac{1}{t} \tag{5.15}$$

Therefore G, p, and ρ vary as $1/t$ and are singular at $t = 0$. Further

$$\rho + p = \frac{1}{8\pi G t^2}\left(\frac{1}{m_2^2} - \frac{1}{m_2} - \frac{1}{m_1 m_2}\right) \tag{5.16a}$$

$$\rho - p = \frac{1}{4\pi G}\left[\frac{1}{t^2}\left(\frac{1}{m_2} - \frac{2}{m_2^2} - \frac{1}{m_1 m_2} - \Lambda_0\right) + \frac{1}{(m_2 D_2 t)^{2/m_2}}\right] \tag{5.16b}$$

$$\rho + 3p = \frac{1}{4\pi G}\left[\frac{1}{t^2}\left(\frac{4}{m_2^2} - \frac{3}{m_2} - \frac{1}{m_1 m_2} + \Lambda_0\right)\frac{1}{(m_2 D_2 t)^{2/m_2}}\right] \tag{5.16c}$$

$$\rho - 3p = \frac{1}{4\pi G}\left[\frac{1}{t^2}\left(\frac{3}{m_2} - \frac{5}{m_2^2} - \frac{1}{m_1 m_2} - 2\Lambda_0\right) + \frac{2}{(m_2 D_2 t)^{2/m_2}}\right] \tag{5.16d}$$

The reality conditions $\rho \geq 0$, $p \geq 0$, and $\rho - 3p \geq 0$ impose further restrictions on the model besides (5.9), (5.12), and (5.13).

6. KANTOWSKI–SACHS MODEL

The Kantowski–Sachs metric is

$$dS^2 = dt^2 - R_1^2(t)\, dr^2 - R_2^2(t)(d\theta^2 + \sin^2\theta\, d\phi^2) \tag{6.1}$$

For the metric (6.1), the field equations (2.1)-(2.2) reduce to

$$2\frac{\ddot{R}_2}{R_2} + \left(\frac{\dot{R}_2}{R_2}\right)^2 + \frac{1}{R_2^2} = 8\pi G p - \Lambda \tag{6.2}$$

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} = 8\pi G p - \Lambda \tag{6.3}$$

$$2\frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} + \left(\frac{\dot{R}_2}{R_2}\right)^2 + \frac{1}{R_2^2} = -8\pi G\rho - \Lambda \tag{6.4}$$

$$8\pi\rho\dot{G} + 8\pi G\left[\dot{\rho} + (\rho + p)\left(\frac{\dot{R}_1}{R_1} + 2\frac{\dot{R}_2}{R_2}\right)\right] + \dot{\Lambda} = 0 \tag{6.5}$$

If we suppose that the energy conservation law holds for matter, then (6.5) reduces to

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{R}_1}{R_1} + 2\frac{\dot{R}_2}{R_2}\right) = 0 \tag{6.6a}$$

$$\dot{\Lambda} = -8\pi\dot{G}\rho \tag{6.6b}$$

We define

$$V(t) = (R_1 R_2^2)^{1/3} \tag{6.7}$$

We assume the solution of equations (6.2)-(6.6) in the form

$$\left.\begin{aligned} V(t) &= (mDt)^{1/m} \\ R_1(t) &= (m_1 D_1 t)^{1/m_1} \\ R_2(t) &= (m_2 D_2 t)^{1/m_2} \\ \Lambda(t) &= \Lambda_0 t^{-2}, \qquad m,\, m_1,\, m_2 \neq 0 \end{aligned}\right\} \tag{6.8}$$

where m, m_1, m_2, D, D_1, D_2, and Λ_0 are constants.

From (6.7) and (6.8), we get

$$m = \frac{3m_1 m_2}{2m_1 + m_2} \tag{6.9}$$

Using (6.8) in (6.2) and (6.4), we get p and ρ, respectively,

$$8\pi G p = \frac{1}{t^2}\left(\Lambda_0 + \frac{3}{m_2^2} - \frac{2}{m_2}\right) + \frac{1}{(m_2 D_2 t)^{2/m_2}} \tag{6.10}$$

$$8\pi G\rho = -\frac{1}{t^2}\left(\Lambda_0 + \frac{1}{m_2^2} + \frac{2}{m_1 m_2}\right) - (m_2 D_2 t)^{-2/m_2} \tag{6.11}$$

From (6.2), (6.3), and (6.8), we get

$$\frac{1}{t^2}\left(\frac{1}{m_1^2}-\frac{1}{m_1}+\frac{1}{m_1 m_2}-\frac{2}{m_2^2}+\frac{1}{m_2}\right)=(m_2 D_2 t)^{-2/m_2} \tag{6.12}$$

This is satisfied and reduces to a relation between the constants when

$$m_2 = 1 \tag{6.13}$$

From (6.6b) and (6.8), we have

$$4\pi\rho\dot{G}=\frac{\Lambda_0}{t^3} \tag{6.14}$$

Equations (6.11) and (6.14) give $\dot{G}/G$. When $G\propto 1/t$, then from (6.10) and (6.11) the pressure and density vary as $1/t$. The model is singular at $t=0$.

We can easily obtain

$$\rho+p=\frac{1}{8\pi G t^2}\left(\frac{1}{m_2^2}-\frac{1}{m_2}-\frac{1}{m_1 m_2}\right) \tag{6.15a}$$

$$\rho-p=\frac{1}{4\pi G}\left[\frac{1}{t^2}\left(\frac{1}{m_2}-\frac{1}{m_1 m_2}-\frac{2}{m_2^2}-\Lambda_0\right)-(m_2 D_2 t)^{-2/m_2}\right] \tag{6.15b}$$

$$\rho+3p=\frac{1}{4\pi G}\left[\frac{1}{t^2}\left(\frac{4}{m_2^2}-\frac{3}{m_2}-\frac{1}{m_1 m_2}+\Lambda_0\right)+(m_2 D_2 t)^{-2/m_2}\right] \tag{6.15c}$$

$$\rho-3p=\frac{1}{4\pi G}\left(\frac{1}{t^2}\left(\frac{3}{m_2}-\frac{5}{m_2^2}-\frac{1}{m_1 m_2}-2\Lambda_0\right)-2(m_2 D_2 t)^{-2/m_2}\right] \tag{6.15d}$$

The reality conditions $\rho\geq 0$, $p\geq 0$, and $\rho-3p\geq 0$ impose further restrictions on the model besides (6.9), (6.12), and (6.13).

7. BIANCHI TYPE V MODEL

The Bianchi type V metric is

$$dS^2=dt^2-R_1^2(t)\,dx^2-e^{-2ax}[R_2^2(t)\,dy^2+R_3^2(t)\,dz^2] \tag{7.1}$$

where $a=\text{const}$.

The field equations (2.1)-(2.2) for the metric (7.1) reduce to

$$\frac{\ddot{R}_2}{R_2}+\frac{\ddot{R}_3}{R_3}+\frac{\dot{R}_2\dot{R}_3}{R_2 R_3}-\frac{a^2}{R_1^2}=8\pi G p-\Lambda \tag{7.2}$$

$$\frac{\ddot{R}_1}{R_1}+\frac{\ddot{R}_3}{R_3}+\frac{\dot{R}_1\dot{R}_3}{R_1 R_3}-\frac{a^2}{R_1^2}=8\pi G p-\Lambda \tag{7.3}$$

$$\frac{\ddot{R}_1}{R_1}+\frac{\ddot{R}_2}{R_2}+\frac{\dot{R}_1\dot{R}_2}{R_1R_2}-\frac{a^2}{R_1^2}=8\pi Gp-\Lambda \tag{7.4}$$

$$\frac{\dot{R}_1\dot{R}_2}{R_1R_2}+\frac{\dot{R}_2\dot{R}_3}{R_2R_3}+\frac{\dot{R}_3\dot{R}_1}{R_3R_1}-\frac{3a^2}{R_1^2}=-8\pi Gp-\Lambda \tag{7.5}$$

$$\frac{\dot{R}_2}{R_2}+\frac{\dot{R}_3}{R_3}-2\frac{\dot{R}_1}{R_1}=0 \tag{7.6}$$

$$8\pi\rho\dot{G}+8\pi G\left[\dot{\rho}+(\rho+p)\left(\frac{\dot{R}_1}{R_1}+\frac{\dot{R}_2}{R_2}+\frac{\dot{R}_3}{R_3}\right)\right]+\dot{\Lambda}=0 \tag{7.7}$$

If we suppose that the energy conservation law holds, then equation (7.7) reduces to

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{R}_1}{R_1}+\frac{\dot{R}_2}{R_2}+\frac{\dot{R}_3}{R_3}\right)=0 \tag{7.8a}$$

$$\dot{\Lambda}=-8\pi\dot{G}\rho \tag{7.8b}$$

We define

$$V(t)=(R_1R_2R_3)^{1/3} \tag{7.9}$$

We assume the solution of equations (7.2)-(7.8) in the form

$$\begin{aligned}
V(t)&=(mDt)^{1/m}\\
R_1(t)&=(m_1D_1t)^{1/m_1}\\
R_2(t)&=(m_2D_2t)^{1/m_2}\\
R_3(t)&=(m_3D_3t)^{1/m_3}\\
\Lambda(t)&=\Lambda_0t^{-2}, \qquad m, m_1, m_2, m_3\neq 0
\end{aligned} \tag{7.10}$$

where m, m_1, m_2, m_3, D, D_1, D_2, D_3, and Λ_0 are arbitrary constants.

From (7.6), (7.9), and (7.10), we get

$$\frac{1}{m}=\frac{1}{3}\left(\frac{1}{m_1}+\frac{1}{m_2}+\frac{1}{m_3}\right) \tag{7.11}$$

$$\frac{1}{m_3}+\frac{1}{m_2}=\frac{3}{m_1} \tag{7.12}$$

Using (7.10) in (7.2) and (7.5), we get the pressure and density,

$$8\pi G p = \frac{1}{t^2}\left(\Lambda_0 + \frac{1}{m_2^2} - \frac{1}{m_2} + \frac{1}{m_3^2} + \frac{1}{m_3} + \frac{1}{m_2 m_3}\right) - a^2(m_1 D_1 t)^{-2/m_1} \tag{7.13}$$

$$8\pi G \rho = -\frac{1}{t^2}\left(\Lambda_0 + \frac{1}{m_1 m_2} + \frac{1}{m_2 m_3} + \frac{1}{m_3 m_1}\right) + 3a^2(m_1 D_1 t)^{-2/m_1} \tag{7.14}$$

From (7.2), (7.3), (7.4), and (7.10), we have

$$\frac{1}{m_2^2} - \frac{1}{m_2} + \frac{1}{m_2 m_3} = \frac{1}{m_1^2} - \frac{1}{m_1} + \frac{1}{m_1 m_3} \tag{7.15}$$

$$\frac{1}{m_3^2} - \frac{1}{m_3} + \frac{1}{m_2 m_3} = \frac{1}{m_1^2} - \frac{1}{m_1} + \frac{1}{m_1 m_2} \tag{7.16}$$

From (7.8b) and (7.10), we get

$$4\pi\rho\dot{G} = \frac{\Lambda_0}{t^3} \tag{7.17}$$

Equations (7.14) and (7.17) give $\dot{G}/G$. If we assume $m_1 = 1$ and $G \propto 1/t$, then from (7.13) and (7.14) the pressure and density vary as $1/t$. The model is singular at $t = 0$.

Further

$$\rho + p = \frac{1}{8\pi G}\left[\frac{1}{t^2}\left(\frac{1}{m_2^2} - \frac{1}{m_2} + \frac{1}{m_3^2} - \frac{1}{m_3} - \frac{1}{m_1 m_2} - \frac{1}{m_1 m_3}\right) + 2a^2(m_1 D_1 t)^{-2/m_1}\right] \tag{7.18a}$$

$$\rho - p = \frac{1}{8\pi G}\left[\frac{1}{t^2}\left(\frac{1}{m_2} - \frac{1}{m_2^2} - \frac{1}{m_3^2} + \frac{1}{m_3} - \frac{1}{m_1 m_2} - \frac{2}{m_2 m_3} - \frac{1}{m_3 m_1} - 2\Lambda_0\right) + 4a^2(m_1 D_1 t)^{-2/m_1}\right] \tag{7.18b}$$

$$\rho + 3p = \frac{1}{8\pi G t^2}\left(2\Lambda_0 + \frac{3}{m_2^2} - \frac{3}{m_2} + \frac{3}{m_3^2} - \frac{3}{m_3} + \frac{2}{m_2 m_3} - \frac{1}{m_1 m_2} - \frac{1}{m_3 m_1}\right) \tag{7.18c}$$

$$\rho - 3p = \frac{1}{8\pi G}\left[\frac{1}{t^2}\left(\frac{3}{m_2} - \frac{3}{m_2^2} - \frac{3}{m_3^2} + \frac{3}{m_3} - \frac{4}{m_2 m_3} - \frac{1}{m_1 m_2} - \frac{1}{m_3 m_1} - 4\Lambda_0\right) + 6a^2(m_1 D_1 t)^{-2/m_1}\right] \tag{7.18d}$$

The reality conditions $\rho \geq 0$, $p \geq 0$, and $\rho - 3p \geq 0$ impose further restrictions on the model besides $m_1 = 1$, (7.15), and (7.16).

8. BIANCHI TYPE VI_0 MODEL

The Bianchi type VI_0 metric is

$$dS^2 = dt^2 - R_1^2(t)\, dx^2 - R_2^2(t) e^{-2ax}\, dy^2 - R_3^2(t) e^{2ax}\, dz^2 \tag{8.1}$$

where $a = \text{const}$.

The field equations (2.1)-(2.2), for the metric (8.1) reduce to

$$\frac{\ddot{R}_2}{R_2} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_2 \dot{R}_3}{R_2 R_3} + \frac{a^2}{R_1^2} = 8\pi G p - \Lambda \tag{8.2}$$

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_3}{R_3} + \frac{\dot{R}_1 \dot{R}_3}{R_1 R_3} - \frac{a^2}{R_1^2} = 8\pi G p - \Lambda \tag{8.3}$$

$$\frac{\ddot{R}_1}{R_1} + \frac{\ddot{R}_2}{R_2} + \frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} + \frac{a^2}{R_1^2} = 8\pi G p - \Lambda \tag{8.4}$$

$$\frac{\dot{R}_1 \dot{R}_2}{R_1 R_2} + \frac{\dot{R}_2 \dot{R}_3}{R_2 R_3} + \frac{\dot{R}_3 \dot{R}_1}{R_3 R_1} - \frac{a^2}{R_1^2} = -8\pi G \rho - \Lambda \tag{8.5}$$

$$\frac{\dot{R}_2}{R_2} - \frac{\dot{R}_3}{R_3} = 0 \tag{8.6}$$

$$8\pi \rho \dot{G} + 8\pi G \left[\dot{\rho} + (\rho + p) \left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3} \right) \right] + \dot{\Lambda} = 0 \tag{8.7}$$

If we suppose that the energy conservation law holds for matter, then (8.7) reduces to

$$\dot{\rho} + (\rho + p) \left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2} + \frac{\dot{R}_3}{R_3} \right) = 0 \tag{8.8a}$$

$$\dot{\Lambda} = -8\pi \rho \dot{G} \tag{8.8b}$$

We define

$$V(t) = (R_1 R_2 R_3)^{1/3} \tag{8.9}$$

We assume the solution of equations (8.2)-(8.8) in the form

$$\begin{aligned} V(t) &= (mDt)^{1/m} \\ R_1(t) &= (m_1 D_1 t)^{1/m_1} \\ R_2(t) &= (m_2 D_2 t)^{1/m_2} \\ R_3(t) &= (m_3 D_3 t)^{1/m_3} \\ \Lambda(t) &= \Lambda_0 t^{-2}, \qquad m, m_1, m_2, m_3 \neq 0 \end{aligned} \tag{8.10}$$

where m, m_1, m_2, m_3, D, D_1, D_2, D_3, and Λ_0 are arbitrary constants. From (8.6), (8.9), and (8.10), we get

$$m_2 = m_3 \tag{8.11}$$

$$\frac{1}{m} = \frac{1}{3}\left(\frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3}\right) \tag{8.12}$$

Using (8.10) in (8.2) and (8.5), we get p and ρ, respectively,

$$8\pi Gp = \frac{1}{t^2}\left(\Lambda_0 + \frac{1}{m_2^2} - \frac{1}{m_2} + \frac{1}{m_3^2} - \frac{1}{m_2 m_3} - \frac{1}{m_3}\right) + a^2(m_1 D_1 t)^{-2/m_1} \tag{8.13}$$

$$8\pi G\rho = -\frac{1}{t^2}\left(\Lambda_0 + \frac{1}{m_1 m_2} + \frac{1}{m_2 m_3} + \frac{1}{m_3 m_1}\right) + a^2(m_1 D_1 t)^{-2/m_1} \tag{8.14}$$

From (8.2)-(8.4) and (8.10), we have

$$\frac{1}{t^2}\left(\frac{1}{m_1^2} - \frac{1}{m_1} + \frac{1}{m_1 m_3} - \frac{1}{m_2^2} + \frac{1}{m_2} - \frac{1}{m_2 m_3}\right) = 2a^2(m_1 D_1 t)^{-2/m_1} \tag{8.15}$$

$$\frac{1}{t^2}\left(\frac{1}{m_1^2} - \frac{1}{m_1} + \frac{1}{m_1 m_2} - \frac{1}{m_3^2} + \frac{1}{m_3} - \frac{1}{m_2 m_3}\right) = 2a^2(m_1 D_1 t)^{-2/m_1} \tag{8.16}$$

The equations are satisfied leading to relations between the constants when $m_1 = 1$.

From (8.8b) and (8.10), we get

$$4\pi\rho\dot{G} = \frac{\Lambda_0}{t^3} \tag{8.17}$$

Equations (8.14) and (8.17) give $\dot{G}/G$. When $m_1 = 1$, $\dot{G}/G \propto 1/t$. When we take $G \propto 1/t$, the pressure and density vary as $1/t$. The model is singular at $t = 0$.

Further

$$\rho + p = \frac{1}{8\pi G}\left[\frac{1}{t^2}\left(\frac{1}{m_2^2} - \frac{1}{m_2} + \frac{1}{m_3^2} - \frac{1}{m_3} - \frac{1}{m_1 m_2} - \frac{1}{m_3 m_1}\right) + 2a^2(m_1 D_1 t)^{-2/m_1}\right] \tag{8.18a}$$

$$\rho - p = \frac{1}{8\pi G t^2}\left(\frac{1}{m_2} - \frac{1}{m_2^2} + \frac{1}{m_3} - \frac{1}{m_3^2} - \frac{1}{m_1 m_2} - \frac{2}{m_2 m_3} - \frac{1}{m_3 m_1} - 2\Lambda_0\right) \tag{8.18b}$$

$$\rho+3p=\frac{1}{8\pi G}\left[\frac{1}{t^2}\left(\frac{3}{m_2^2}-\frac{3}{m_2}+\frac{3}{m_2^2}-\frac{3}{m_3}+\frac{2}{m_2m_3}-\frac{1}{m_3m_1}-\frac{1}{m_1m_2}+2\Lambda_0\right)\right.$$
$$+4a^2(m_1D_1t)^{-2/m_1} \tag{8.18c}$$

$$\rho-3p=\frac{1}{8\pi G}\left[\left(\frac{1}{t^2}\frac{3}{m_2}-\frac{3}{m_2^2}-\frac{3}{m_3^2}+\frac{3}{m_3}-\frac{1}{m_1m_2}-\frac{4}{m_3m_2}-\frac{1}{m_3m_1}-4\Lambda_0\right)\right.$$
$$\left.-2a^2(m_1D_1t)^{-2/m_1}\right] \tag{8.18d}$$

The reality conditions $\rho\geq 0$, $p\geq 0$, and $\rho-3p\geq 0$ put restrictions on the model.

9. BIANCHI TYPE VIII MODEL

The Bianchi type VIII metric is

$$dS^2=dt^2-S^2\,dx^2-R^2\,dy^2-(R^2\sinh^2 y+S^2\cosh^2 y)\,dz^2$$
$$-2S^2\cosh y\,dx\,dz \tag{9.1}$$

where $S=S(t)$, $R=R(t)$.

The field equations (2.1)-(2.2) for the metric (9.1) reduce to

$$2\frac{\ddot{R}}{R}+\left(\frac{\dot{R}}{R}\right)^2-\frac{1}{R^2}-\frac{3}{4}\frac{S^2}{R^4}=8\pi Gp-\Lambda \tag{9.2}$$

$$\frac{\ddot{R}}{R}+\frac{\ddot{S}}{S}+\frac{\dot{R}\dot{S}}{RS}+\frac{1}{4}\frac{S^2}{R^4}=8\pi Gp-\Lambda \tag{9.3}$$

$$2\frac{\dot{R}\dot{S}}{RS}+\left(\frac{\dot{R}}{R}\right)^2-\frac{1}{R^2}-\frac{1}{4}\frac{S^2}{R^4}=-8\pi\rho G-\Lambda \tag{9.4}$$

$$8\pi\rho\dot{G}+8\pi G\left[\dot{\rho}+(\rho+p)\left(\frac{\dot{S}}{S}+\frac{2\dot{R}}{R}\right)\right]+\dot{\Lambda}=0 \tag{9.5}$$

If we assume that the energy conservation law holds for matter, then (9.5) reduces to

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{S}}{S}+\frac{2\dot{R}}{R}\right)=0 \tag{9.6a}$$

$$\dot{\Lambda}=-8\pi\dot{G}\rho \tag{9.6b}$$

We define

$$V(t)=(SR^2)^{1/3} \tag{9.7}$$

We assume the solution of equations (9.2)-(9.6) in the form

$$\begin{aligned} V(t) &= (mDt)^{1/m} \\ S(t) &= (m_1 D_1 t)^{1/m_1} \\ R(t) &= (m_2 D_2 t)^{1/m_2} \\ \Lambda(t) &= \Lambda_0 t^{-2}, \qquad m, m_1, m_2 \neq 0 \end{aligned} \tag{9.8}$$

where m, m_1, m_2, D, D_1, D_2, and Λ_0 are constants.

From (9.7) and (9.8), we get

$$m = \frac{3m_1 m_2}{2m_1 + m_2} \tag{9.9}$$

Using (9.8) in (9.2) and (9.4), we get p and ρ, respectively,

$$\begin{aligned} 8\pi G p = {} & \frac{1}{t^2}\left(\Lambda_0 + \frac{3}{m_2^2} - \frac{2}{m_2}\right) - (m_2 D_2 t)^{-2/m_2} \\ & - \frac{3}{4}\frac{(m_1 D_1 t)^{2/m_1}}{(m_2 D_2 t)^{4/m_2}} \end{aligned} \tag{9.10}$$

$$\begin{aligned} 8\pi G \rho = {} & -\frac{1}{t^2}\left(\Lambda_0 + \frac{2}{m_1 m_2} + \frac{1}{m_2^2}\right) + \frac{1}{(m_2 D_2 t)^{2/m_2}} \\ & + \frac{1}{4}\frac{(m_1 D_1 t)^{2/m_1}}{(m_2 D_2 t)^{4/m_2}} \end{aligned} \tag{9.11}$$

From (9.2), (9.3), and (9.8), we have

$$\begin{aligned} & \frac{1}{t^2}\left(\frac{2}{m_2^2} - \frac{1}{m_2} - \frac{1}{m_1^2} + \frac{1}{m_1} - \frac{1}{m_1 m_2}\right) \\ & \quad = (m_2 D_2 t)^{-2/m_2} + \frac{(m_1 D_1 t)^{2/m_1}}{(m_2 D_2 t)^{4/m_2}} \end{aligned} \tag{9.12}$$

This is satisfied and reduces to a relation between the constants when

$$m_1 = m_2 = 1 \tag{9.13}$$

From (9.6b) and (9.8), we have

$$4\pi \rho \dot{G} = \frac{\Lambda_0}{t^3} \tag{9.14}$$

Equations (9.11) and (9.14) give $\dot{G}/G$. If we take $G\propto 1/t$ and assume that (9.13) holds, then the pressure and density vary as $1/t$. The model is singular at $t=0$.

Further

$$\rho+p=\frac{1}{4\pi G}\left[\frac{1}{t^2}\left(\frac{1}{m_2^2}-\frac{1}{m_2}-\frac{1}{m_1 m_2}\right)-\frac{1}{4}\frac{(m_1 D_1 t)^{2/m_1}}{(m_2 D_2 t)^{4/m_2}}\right] \tag{9.15a}$$

$$\rho-p=\frac{1}{4\pi G}\left[-\frac{1}{t^2}\left(\Lambda_0+\frac{1}{m_1 m_2}+\frac{2}{m_2^2}-\frac{1}{m_2}\right)+(m_2 D_2 t)^{-2/m_2}+\frac{1}{2}\frac{(m_1 D_1 t)^{2/m_1}}{(m_2 D_2 t)^{4/m_2}}\right] \tag{9.15b}$$

$$\rho+3p=\frac{1}{4\pi G}\left[\frac{1}{t^2}\left(\Lambda_0-\frac{1}{m_1 m_2}+\frac{4}{m_2^2}-\frac{3}{m_2}\right)-(m_2 D_2 t)^{-2/m_2}-\frac{(m_1 D_1 t)^{2/m_1}}{(m_2 D_2 t)^{4/m_2}}\right] \tag{9.15c}$$

$$\rho-3p=\frac{1}{4\pi G}\left[\frac{1}{t^2}\left(\frac{3}{m_2}-\frac{1}{m_1 m_2}-\frac{5}{m_2^2}-2\Lambda_0\right)+2(m_2 D_2 t)^{-2/m_2}+\frac{5}{4}\frac{(m_1 D_1 t)^{2/m_1}}{(m_2 D_2 t)^{4/m_2}}\right] \tag{9.15d}$$

The reality conditions $\rho\geq 0$, $p\geq 0$, and $\rho-3p\geq 0$ impose further restrictions on the model.

10. BIANCHI TYPE IX MODEL

The Bianchi type IX metric is

$$dS^2=dt^2-S^2\,dx^2-R^2\,dy^2-(R^2\sin^2 y+S^2\cos^2 y)\,dz^2+2S^2\cos y\,dx\,dz \tag{10.1}$$

where

$$S=S(t),\qquad R=R(t)$$

The field equations (2.1)-(2.2) for the metric (10.1) reduce to

$$2\frac{\ddot{R}}{R}+\left(\frac{\dot{R}}{R}\right)^2+\frac{1}{R^2}-\frac{3}{4}\frac{S^2}{R^4}=8\pi Gp-\Lambda \tag{10.2}$$

$$\frac{\ddot{R}}{R}+\frac{\ddot{S}}{S}+\frac{\dot{R}\dot{S}}{RS}+\frac{1}{4}\frac{S^2}{R^4}=8\pi Gp-\Lambda \tag{10.3}$$

$$2\frac{\dot{R}\dot{S}}{RS}+\left(\frac{\dot{R}}{R}\right)^2+\frac{1}{R^2}-\frac{1}{4}\frac{S^2}{R^4}=-8\pi G\rho-\Lambda \tag{10.4}$$

$$8\pi\rho\dot{G}+8\pi G\left[\dot{\rho}+(\rho+p)\left(\frac{\dot{S}}{S}+2\frac{\dot{R}}{R}\right)\right]+\dot{\Lambda}=0 \tag{10.5}$$

If we assume that the energy conservation law holds for matter, (10.5) reduces to

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{S}}{S}+2\frac{\dot{R}}{R}\right)=0 \tag{10.6a}$$

$$\dot{\Lambda}=-8\pi\dot{G}\rho \tag{10.6b}$$

We define

$$V(t)=(SR^2)^{1/3} \tag{10.7}$$

We assume the solution of equations (10.2)-(10.6) in the form

$$\begin{aligned} V(t)&=(mDt)^{1/m}\\ S(t)&=(m_1D_1t)^{1/m_1}\\ R(t)&=(m_2D_2t)^{1/m_2}\\ \Lambda(t)&=\Lambda_0t^{-2}, \qquad m,\, m_1,\, m_2\neq 0, \end{aligned} \tag{10.8}$$

where m, m_1, m_2, D, D_1, D_2, and Λ_0 are arbitrary constants.

From (10.7) and (10.8), we have

$$m=\frac{3m_1m_2}{2m_1+m_2} \tag{10.9}$$

Using (10.8) in (10.2) and (10.4), we get p and ρ, respectively,

$$\begin{aligned} 8\pi Gp&=\frac{1}{t^2}\left(\Lambda_0+\frac{3}{m_2^2}-\frac{2}{m_2}\right)+(m_2D_2t)^{-2/m_2}\\ &\quad-\frac{3}{4}\frac{(m_1D_1t)^{2/m_1}}{(m_2D_2t)^{4/m_2}} \end{aligned} \tag{10.10}$$

$$\begin{aligned} 8\pi G\rho&=-\frac{1}{t^2}\left(\Lambda_0+\frac{2}{m_1m_2}+\frac{1}{m_2^2}\right)-(m_2D_2t)^{-2/m_2}\\ &\quad+\frac{1}{4}\frac{(m_1D_1t)^{2/m_1}}{(m_2D_2t)^{4/m_2}} \end{aligned} \tag{10.11}$$

From (10.2), (10.3), and (10.8), we get

$$\frac{1}{t^2}\left(\frac{1}{m_1^2}-\frac{1}{m_1}+\frac{1}{m_1 m_2}-\frac{2}{m_2^2}+\frac{1}{m_2}\right)$$
$$=(m_2 D_2 t)^{-2/m_2}-\frac{(m_1 D_1 t)^{2/m_1}}{(m_2 D_2 t)^{4/m_2}} \tag{10.12}$$

This is satisfied and leads to a relation between the constants when

$$m_1 = m_2 = 1 \tag{10.13}$$

From (10.6b) and (10.8), we get

$$4\pi\rho\dot{G}=\frac{\Lambda_0}{t^3} \tag{10.14}$$

From (10.11) and (10.14) we can obtain $\dot{G}/G$. If we assume that $G \propto 1/t$ and also that (10.13) holds, then from (10.10) and (10.11) the pressure and density vary as $1/t$. The model is singular at $t=0$.

Also

$$\rho+p=\frac{1}{4\pi G}\left[\frac{1}{t^2}\left(\frac{1}{m_2^2}-\frac{1}{m_2}-\frac{1}{m_1 m_2}\right)\right.$$
$$\left.-\frac{1}{4}\frac{(m_1 D_1 t)^{2/m_1}}{(m_2 D_2 t)^{4/m_2}}\right] \tag{10.15a}$$

$$\rho-p=\frac{1}{4\pi G}\left[-\frac{1}{t^2}\left(\Lambda_0+\frac{1}{m_1 m_2}+\frac{2}{m_2^2}-\frac{1}{m_2}\right)\right.$$
$$\left.-(m_2 D_2 t)^{-2/m_2}+\frac{1}{2}\frac{(m_1 D_1 t)^{2/m_1}}{(m_2 D_2 t)^{4/m_2}}\right] \tag{10.15b}$$

$$\rho+3p=\frac{1}{4\pi G}\left[\frac{1}{t^2}\left(\Lambda_0-\frac{1}{m_1 m_2}+\frac{4}{m_2^2}-\frac{3}{m_2}\right)\right.$$
$$\left.+(m_2 D_2 t)^{-2/m_2}-\frac{(m_1 D_1 t)^{2/m_1}}{(m_2 D_2 t)^{4/m_2}}\right] \tag{10.15c}$$

$$\rho-3p=\frac{1}{4\pi G}\left[\frac{1}{t^2}\left(\frac{3}{m_2}-\frac{1}{m_1 m_2}-\frac{5}{m_2^2}-2\Lambda_0\right)\right.$$
$$\left.-2(m_2 D_2 t)^{-2/m_2}+\frac{5}{4}\frac{(m_1 D_1 t)^{2/m_1}}{(m_2 D_2 t)^{4/m_2}}\right] \tag{10.15d}$$

The reality conditions $\rho \geq 0$, $p \geq 0$, and $\rho - 3p \geq 0$ impose further restrictions on the model.

11. CONCLUSIONS

We have investigated Bianchi-type models in which the cosmological and gravitational constants vary with time. The Hubble parameter is assumed to follow a power-law variation with time and $\Lambda \propto t^{-2}$. All the models start from a singular state at the epoch $t=0$. The gravitational constant G can be a decreasing or increasing function of time. For $G \propto 1/t$, the pressure and density show a simple behavior and decrease with time. The cosmological constant Λ is gradually reduced as the universe expands.

REFERENCES

Abers, E. S., and Lee, B. W. (1973). *Physics Reports*, **9**, 1.

Abdel-Rahman, A. M. M. (1990). *General Relativity and Gravitation*, **22**, 655.

Beesham, A. K. (1986). *International Journal of Theoretical Physics*, **25**, 1295.

Beesham, A. K. (1988). Ph.D. Thesis, University of Cape Town, South Africa.

Berman, M. S. (1983). *Nuovo Cimento B*, **74**, 182.

Berman, M. S. (1988). *General Relativity and Gravitation*, **20**, 841.

Berman, M. S. (1990*a*). *International Journal of Theoretical Physics*, **29**, 567.

Berman, M. S. (1990*b*). *Nuovo Cimento B*, **105**, 239.

Berman, M. S. (1991*a*). *Physical Review D*, **43**, 1075.

Berman, M. S. (1991*b*). Preprint.

Berman, M. S., and Gomide, F. M. (1988). *General Relativity and Gravitation*, **20**, 191.

Berman, M. S., and Som, M. M. (1990). *General Relativity and Gravitation*, **22**, 625.

Berman, M. S., Som, M. M., and Gomide, F. M. (1989). *General Relativity and Gravitation*, **21**, 287.

Bertolami, O. (1986*a*). *Nuovo Cimento B*, **93**, 36.

Bertolami, O. (1986*b*). *Fortschritte der Physik*, **34**, 829.

Chen, W., and Wu, Y. S. (1990). *Physical Review D*, **41**, 695.

Langacker, P. (1981). *Physics Reports*, **72**, 185.

Wahba, Renee R. (1989). Ph.D. Thesis, University of Alabama, Huntsville, Alabama.

Weinberg, S. (1972). *Gravitation and Cosmology*, Wiley, New York.